# Hydropower Plants: Generating and Pumping Units
# Solved Problems: Series 1

## 1 ENERGY LOSS CALCULATION

Consider a piping system from a dam to a hydropower plant (see Figure 1) including fittings and valves. Answer to the questions, using the values provided in Figure 1 and the information from appendices A and B. The gravity acceleration and water kinematic viscosity are $g = 9.81$ ms$^{-2}$ and $\nu_{water} = 10^{-6}\ \mathrm{m^2\,s^{-1}}$.

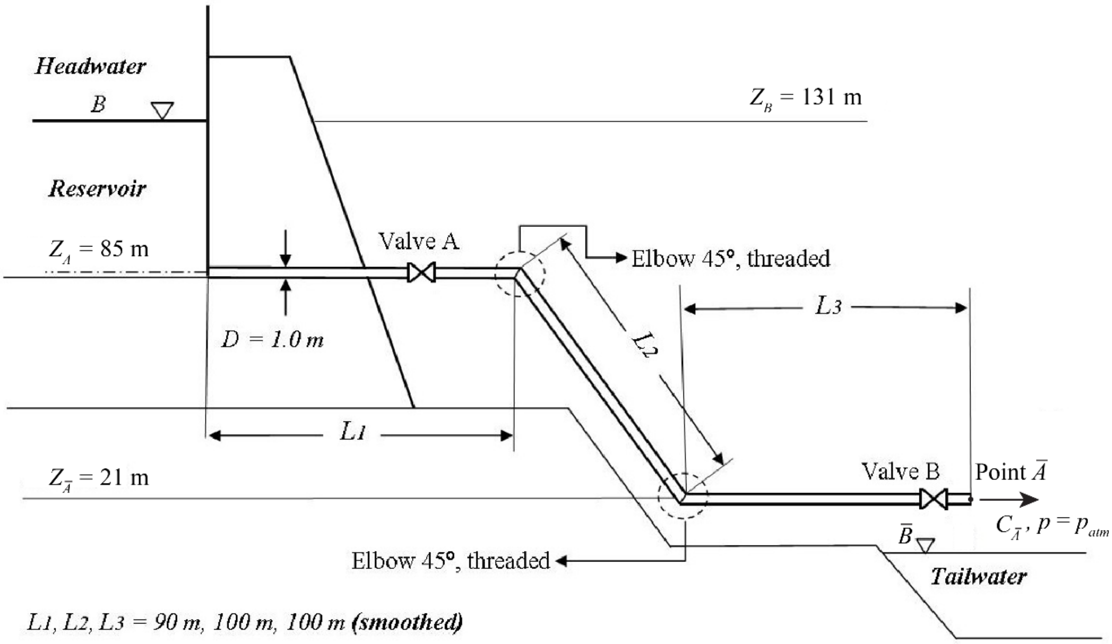


*Figure 1: Dam piping system*

1) Derive a relation between velocity $C_{\bar{A}}$ and head $Z_B - Z_{\bar{A}}$ based on energy balance, by i) neglecting and ii) considering energy losses $gH_{rB\div\bar{A}}$:

*Considering the specific energy balance equation and specific energy losses $gH_{rB\div\bar{A}}$ between the headwater level B and point $\bar{A}$, we can derive the following equation:*

$$\frac{P_B}{\rho} + \frac{C_B^2}{2} + gZ_B = \frac{P_{\bar{A}}}{\rho} + \frac{C_{\bar{A}}^2}{2} + gZ_{\bar{A}} + gH_{rB\div\bar{A}} \qquad (1)$$

*Knowing that $C_B = 0$ and $P_B = P_{\bar{A}} = P_{atm}$:*

*i)* $C_{\overline{A}} = \sqrt{2g\left(Z_B - Z_{\overline{A}}\right) - 2gH_{rB\div\overline{A}}}$

*ii)* *By neglecting specific energy losses,* $gH_{rB\div 1} = 0$ *:*

$$C_{\overline{A}\_without\_Loss} = \sqrt{2g\left(Z_B - Z_{\overline{A}}\right)}$$

2) Calculate the velocity $C_{\overline{A}\_without\_Loss}$ at the point $\overline{A}$ and the discharge $Q$ by assuming all the specific energy losses are negligible.

*Using the expression derived in 1):*

$$C_{\overline{A}\_without\_Loss} = \sqrt{2g\left(Z_B - Z_{\overline{A}}\right)} = 46.46 \text{ m s}^{-1}$$

*Thus, using the relation between discharge, section and velocity* $Q = CA$ *:*

$$Q = C_{\overline{A}\_without\_Loss} \times \frac{\pi D^2}{4} = 46.46 \times \frac{3.1415 \times 1.0^2}{4} = 36.49 \text{ m}^3 \text{ s}^{-1}$$

3) Calculate the Reynolds number neglecting the specific energy losses.

$$\mathrm{Re} = \frac{C_{\overline{A}\_without\_Loss} D}{\nu_{water}} = \frac{\sqrt{2g\left(Z_B - Z_{\overline{A}}\right)} D}{\nu_{water}} = 4.6 \times 10^7$$

4) The actual discharge $Q$ is $13.66 \text{ m}^3 \text{ s}^{-1}$. Compute the singular and distributed specific energy losses, $gH_{rB\div\overline{A}\_\text{singular}}$ and $gH_{rB\div\overline{A}\_distributed}$. Use *Figure A.1* to find the regular specific energy losses local coefficient $\lambda$. Consider the surface of the piping system as perfectly smooth, and assume the valves *A* and *B* as gate valves, respectively fully open and ½ closed.

*The actual velocity is found from the actual discharge:*

$$C_{with\_Loss} = \frac{4Q}{\pi D^2} = 17.39 \text{ m s}^{-1}$$

*Then, the energy losses coefficient is found by applying the Reynolds number to the smooth curve from Figure A.1:*

$$\mathrm{Re} = \frac{CD}{\nu_{water}} = 1.7 \times 10^7 \quad \xrightarrow{from\ figure\ A.1} \quad \lambda \approx 0.0075$$

*And the distributed specific energy losses are therefore:*

$$gH_{rB\div\overline{A}\_distributed} = \frac{C^2_{with\_loss}}{2} \lambda \frac{\sum L}{D} = \frac{C^2_{with\_loss}}{2} \lambda \frac{L1 + L2 + L3}{D} = 328.87 \text{ J kg}^{-1}$$

*Using the specific energy losses coefficients k listed in Table B.1, the singular specific losses are:*

$$gH_{rB\div\overline{A}\_\text{singular}} = \frac{C^2_{with\_Loss}}{2}\left(k_{intake} + k_{valve\ A} + 2k_{elbow} + k_{valve\ B}\right) = \frac{17.39^2}{2}\left(0.5 + 0.15 + 2 \cdot 0.40 + 2.10\right) = 536.96 \text{ J kg}^{-1}$$

5) If the penstock diameter is increased to 1.2 m, compute the new regular and singular specific energy losses.

*Following the exact same methodology as in 4):*

$$C_{new} = \frac{4Q}{\pi D_{new}^{2}} = 12.08\,\mathrm{m\,s^{-1}}$$

$$\mathrm{Re}_{new} = \frac{C_{new} D_{new}}{\nu_{water}} = 1.4 \times 10^{7} \quad \xrightarrow{\text{from figure A.1}} \quad \lambda_{new} \approx 0.0078$$

$$gH_{rB \div \bar{A}\_\mathrm{singular}} = \frac{C_{new}^{2}}{2}\left(k_{intake} + k_{valve\ A} + 2k_{elbow} + k_{valve\ B}\right) = 259.01\ \mathrm{J\,kg^{-1}}$$

$$gH_{rB \div \bar{A}\_distributed} = \frac{C_{new}^{2}}{2}\lambda_{new}\frac{\sum L}{D_{new}} = 137.54\ \mathrm{J\,kg^{-1}}$$

6) This time, compute the regular and singular specific energy losses if the penstock diameter is reduced to 0.8 m.

*Once again following the previous methodology:*

$$C_{small} = \frac{4Q}{\pi D_{small}^{2}} = 27.17\,\mathrm{m\,s^{-1}}$$

$$\mathrm{Re}_{small} = \frac{C_{small} D_{small}}{\nu_{water}} = 2.2 \times 10^{7} \quad \xrightarrow{\text{from figure A.1}} \quad \lambda_{small} \approx 0.0072$$

$$gH_{rB \div \bar{A}\_\mathrm{singular}} = \frac{C_{small}^{2}}{2}\left(k_{intake} + k_{valve\ A} + 2k_{elbow} + k_{valve\ B}\right) = 1310.32\,\mathrm{J\,kg^{-1}}$$

$$gH_{rB \div \bar{A}\_distributed} = \frac{C_{small}^{2}}{2}\lambda_{small}\frac{\sum L}{D_{small}} = 963.36\,\mathrm{J\,kg^{-1}}$$

# 2 GENERAL HYDRAULIC POWER PLANT

## 2.1 Basic calculation for a hydraulic power plant

In Figure 2, the sketch of a hydraulic power plant located in Brazil is shown. The elevations of the headwater and tailwater reservoirs are $Z_B = 304$ m and $Z_{\bar{B}} = 252$ m, respectively. The rated discharge is $Q$ = 539 m$^3$ s$^{-1}$ and the global efficiency $\eta$ is 0.91. If necessary, use the following values of gravity acceleration and water density:

$g$ = 9.81 m s$^{-2}$, $\rho$ = 1'000 kg m$^{-3}$

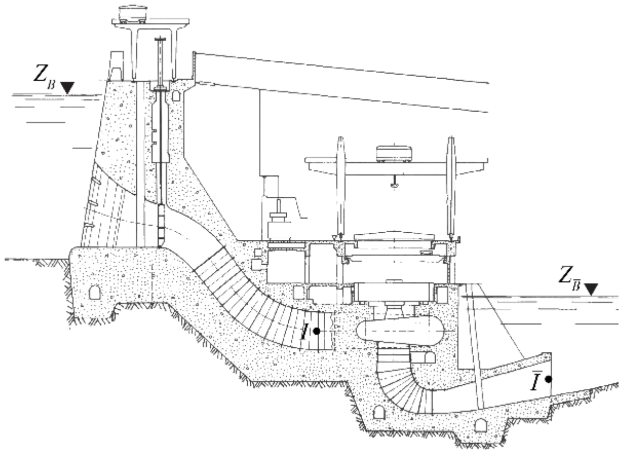


*Figure 2 Sketch of the hydraulic power plant and the detail of the turbine*

7) Express the potential specific energy of the installation by $Z_B$, $Z_{\bar{B}}$, and $g$.

*The potential specific energy of the installation is* $g(Z_B - Z_{\bar{B}})$

8) Taking into account the specific energy losses $gH_{rB \div I}$ between $B$ and $I$, and $gH_{r\bar{I} \div \bar{B}}$, between $\bar{I}$ and $\bar{B}$ ($gH_r > 0$), express the available specific energy $E$ by $g$, $Z_B$, $Z_{\bar{B}}$, $gH_{rB \div I}$ and $gH_{r\bar{I} \div \bar{B}}$.

*The available specific energy can be written as* $E = g(Z_B - Z_{\bar{B}}) - gH_{rB \div I} - gH_{r\bar{I} \div \bar{B}}$

9) Calculate the water velocity in the penstock. Use the value of the penstock diameter $D_{penstock}$ = 7 m.

*Using the same methodology as in Exercise 1, the water velocity in the penstock is:*

$$C_{penstock} = \frac{4Q}{\pi D_{penstock}^2} \cong 14.01 \, \mathrm{m\ s^{-1}}$$

10) Calculate the Reynolds number in the penstock using the kinetic viscosity $\nu = 10^{-6}$ m$^2$ s$^{-1}$.

*The Reynolds number in the penstock is* $\mathrm{Re}_{penstock} = \dfrac{C_{penstock} \cdot D_{penstock}}{\nu} \cong 9.80 \times 10^7$

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## 2.2 Practical study for the specific energy loss

Here, the specific energy loss calculation is applied to the practical case for the hydraulic power station detailed in *Section 2.1.*

11) Knowing that the regular specific energy loss in a penstock can be expressed as:

$$gH_{r1 \div 2} = K_r \frac{C^2}{2} = \lambda \frac{L_{1 \div 2}}{D} \frac{C^2}{2} \tag{1}$$

Express the distributed specific energy loss $gH_r$ as a function of the local coefficient of the distributed specific losses $\lambda$, the length of the penstock $L_{penstock}$, the diameter of the penstock $D_{penstock}$ and the discharge $Q$.

Then, reflect on the importance of the penstock diameter and explain why it is a key parameter to reduce the specific energy loss under a constant discharge.

$$gH_r = \lambda \frac{L_{penstock}}{D_{penstock}} \frac{C^2_{penstock}}{2} = \lambda \frac{8 L_{penstock} Q^2}{\pi D^5_{penstock}}$$

*The losses are a function of the penstock diameter to the power of 5 (!) Thus, for the same discharge, doubling the diameter of the penstock would divide the losses by a factor of 32. However, increasing the penstock diameter induces higher construction costs.*

12) The local coefficient of the distributed specific energy losses $\lambda$ is dependent on the Reynolds number Re and can be calculated by the Churchill formula as follows:

$$\lambda = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + \frac{1}{(A+B)^{\frac{3}{2}}}\right]^{\frac{1}{12}}$$

$$\text{with } A = \left[2.457 \cdot \ln \frac{1}{\left(\frac{7}{\mathrm{Re}}\right)^{0.9} + 0.27 \frac{k_s}{D_{penstock}}}\right]^{16} \text{ and } B = \left(\frac{37530}{\mathrm{Re}}\right)^{16} \qquad (2)$$

Where $k_s$ is the equivalent sand roughness, whose value which depends on the penstock material. For instance, $k_s = 10^{-6}$ for stainless steel and $k_s = 3 \times 10^{-3}$ for rough concrete.

Calculate the local coefficient values $\lambda_{steel}$ and $\lambda_{concrete}$ when using i) stainless steel and ii) rough concrete as the penstock material. Then, calculate the distributed specific energy loss in the penstock for both cases. Use the penstock length $L_{penstock} = 100$ m.

$A_{steel} = 8.4809 \times 10^{24}$, $B_{steel} = 2.1264 \times 10^{-55}$, *and* $\lambda_{steel} = 0.0061$

$A_{concrete} = 3.6429 \times 10^{21}$, $B_{steel} = 2.1264 \times 10^{-55}$, *and* $\lambda_{concrete} = 0.0161$

*The distributed specific energy losses are thus:*

$$i)\ gH_{rB \div I} = \lambda_{steel} \frac{L_{penstock}}{D_{penstock}} \frac{C^2_{penstock}}{2} = 8.58 \text{ J kg}^{-1}$$

$$ii)\ gH_{rB \div I} = \lambda_{concrete} \frac{L_{penstock}}{D_{penstock}} \frac{C^2_{penstock}}{2} = 22.61 \text{ J kg}^{-1}$$

*Which means that building the penstock with concrete results in 3 times more losses than with stainless steel.*

13) Assuming that the total specific energy losses of the singular specific energy losses (intake, elbow, etc…) and the specific energy losses $gH_{r\bar{I} \div \bar{B}}$ are equivalent to 1% of the gross head, calculate the available specific energy $E$ for both cases.

*Using the fact that* $gH_{r\bar{I} \div \bar{B}}$ *can simply be expressed as a 1% reduction of the gross head, meaning a 1% reduction in specific potential energy, and the values of the distributed energy losses computed in 12):*

$$E = g(Z_B - Z_{\bar{B}}) - gH_{rB \div I} - gH_{r\bar{I} \div \bar{B}} = 0.99 g (Z_B - Z_{\bar{B}}) - \lambda \frac{L_{penstock}}{D_{penstock}} \frac{C^2_{penstock}}{2}$$

*Which results in:*

*i)* $E = 496.44\ J\ kg^{-1}$

*ii)* $E = 482.41\ J\ kg^{-1}$

14) For both cases, calculate the available power $P$ and compare the difference between both cases.

*The available power is defined as* $P = \eta\rho QE$*, with* $\eta$ *the global efficiency given in Section 2.1. Thus:*
*i)* $P = 242.8\ MW$

*ii)* $P = 235.9\ MW$

*The power difference is thus* $\Delta P = 6.9\ MW$*, which represents 2.8%.*

15) In this power plant, the grid frequency is $f_{grid} = 60$ Hz and the number of poles is $z_p = 88$. Deduce the angular rotational frequency of the runner $\omega$.

*Using the definition of the angular rotational velocity:*

$$\omega = 2\pi n = 2\pi \frac{2 f_{grid}}{z_p} \cong 8.57\,\mathrm{rad\ s^{-1}}.$$

# Appendix A

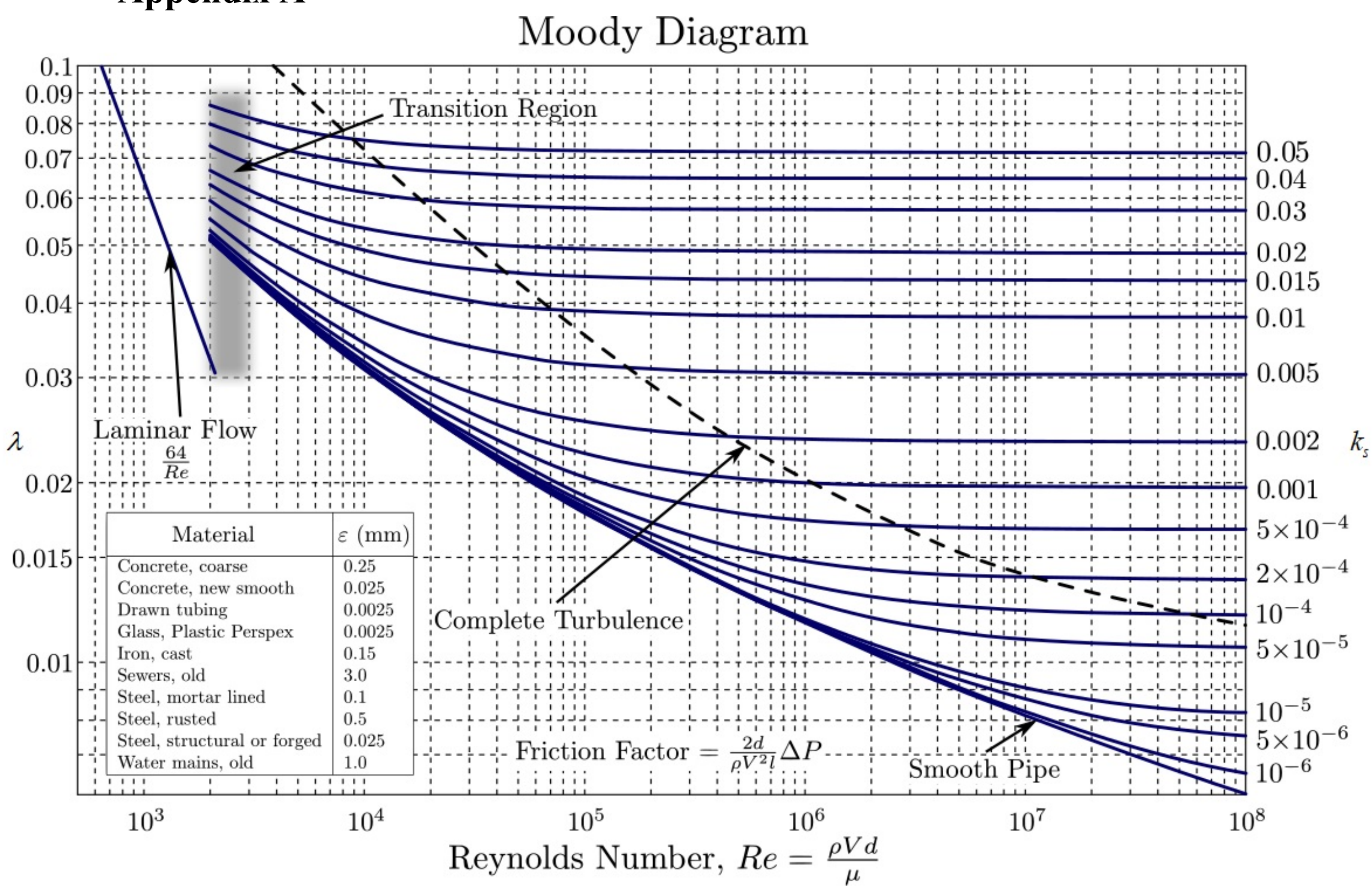


*Figure A.1: Distributed specific energy losses local coefficient for pipe flow*

# Appendix B

*Table B.1: Specific energy loss coefficients of bends, elbows, fittings, etc.*

| Fitting | $k$ [-] |
|---|---|
| Sharp intake connection | 0.5 |
| Globe valve, fully open | 10.0 |
| Angle valve, fully open | 2.0 |
| Gate valve, fully open | 0.15 |
| Gate valve, 1/2 closed | 2.10 |
| Swing check valve, flow | 2.0 |
| Elbow 90° – flanged | 0.3 |
| Elbow 90° – threaded | 1.50 |
| Long radius 90°, flanged | 0.20 |
| Long radius 90°, threaded | 0.70 |
| Elbow 45°, threaded | 0.40 |